Instruction Manual for the Retirement Probability Analyzer

Written and created by:
Moshe A. Milevsky and Anna Abaimova

The IFID Centre

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Operating Systems

To operate effectively, it is strongly recommended that the *Retirement Probability Analyzer* program is used with one of the following systems: the new Windows XP or Windows 2000. These two systems are most appropriate because they are equipped with dynamic link library (DLL) files, which the program relies upon.

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Background

This program allows the user to explore in great detail a technical problem that has occupied a number of investors, financial writers and especially retirees, during the last few years, as the baby boomers start to approach retirement. Namely, what is the probability that one will *run out of money* before *running out of life*, if his or her current standard of living is maintained?

Indeed, the question has even reached the popular mainstream press. For example, a recent article in *Money Magazine* (May 2004) discussed the probability of running out of money – or the odds of being ruined – assuming an investor selected a particular asset allocation (investment portfolio) and product allocation (mixture of fixed and variable annuities). The reported probabilities in that article were all based on Monte Carlo computer simulations. This program takes a distinct approach.

The purchase of payout (also called life or pension) annuities provides a form of longevity insurance, to cover the risk of living too long and running out of money; and in exchange for a lump sum of money, the issuing insurance company provides the investor with a regular income stream, elected to be fixed or variable, for the rest of his or her life.

This program – the *Retirement Probability Analyzer* (RetPro) - consists of a collection of sequential *modules* – written specifically for the Society of Actuaries – that compute probabilities associated with various retirement investment and spending strategies. The underlying and main idea behind the program is to quantify the odds that a particular self-annuitization pattern (i.e. involving no reliance on purchased payout annuities) is sustainable given a specific initial age, mortality table and portfolio asset allocation. To this end, we have defined the Lifetime Probability of Ruin as the probability that the retiree will exhaust all discretionary funds while still alive, if they decide not to annuitize. Clearly, the larger this probability, the greater the argument in favor of annuitization (or reducing consumption and spending).

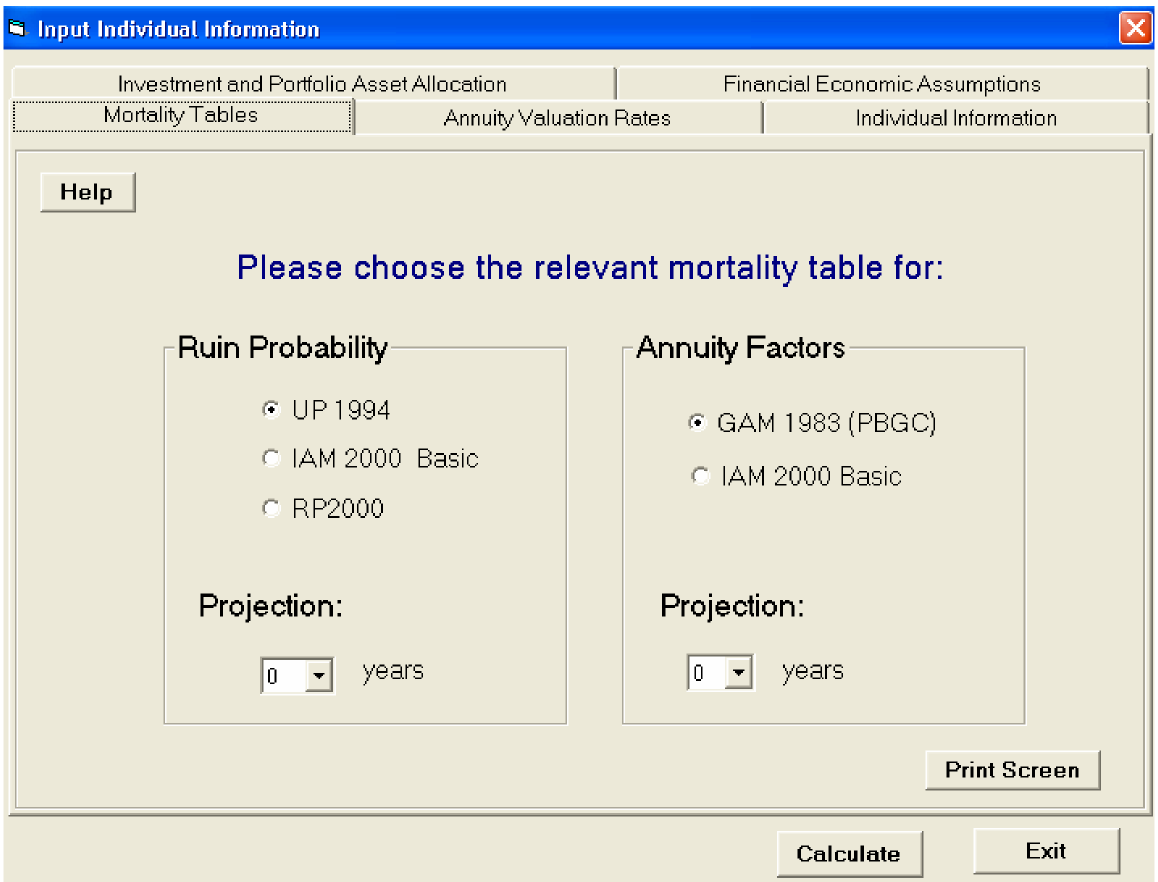
The core of this program does not rely on computer simulations. Rather, it uses a numerical procedure based on the underlying asset dynamics to derive the relevant probabilities and estimates. The benefit in our approach is that our results do not contain statistical sampling error. More importantly, our quick and efficient algorithm allows the user to test any number of cases to obtain precise results for that scenario. Another difference between our approach and the simulation methodology that other authors and practitioners use is that we allow for a richer menu of payout annuity products, such as real inflation linked annuities, variable annuities with flexible assumed interest rates, and nominal annuities with escalating COLA payments.

Overall, we believe that our methodology – which we label the Numerical PDE approach and is explained in greater depth in the technical appendix – provides the user with many degrees of freedom to explore the impact of a variety of retirement income choices.

In the following pages, we will illustrate the various components and capabilities of the program and will apply the methodology in a series of cases. Finally, we provide a Frequently Asked Questions section and a technical appendix in which we address questions the reader may have about our approach of assessing retirement strategies.

Part 1 – Input of Individual Information

The initial phase of the program requires the user to select the mortality table of choice, to specify the assumed yield curve and input the characteristics of the individual, as well as the investment and consumption strategy.



Mortality Tables

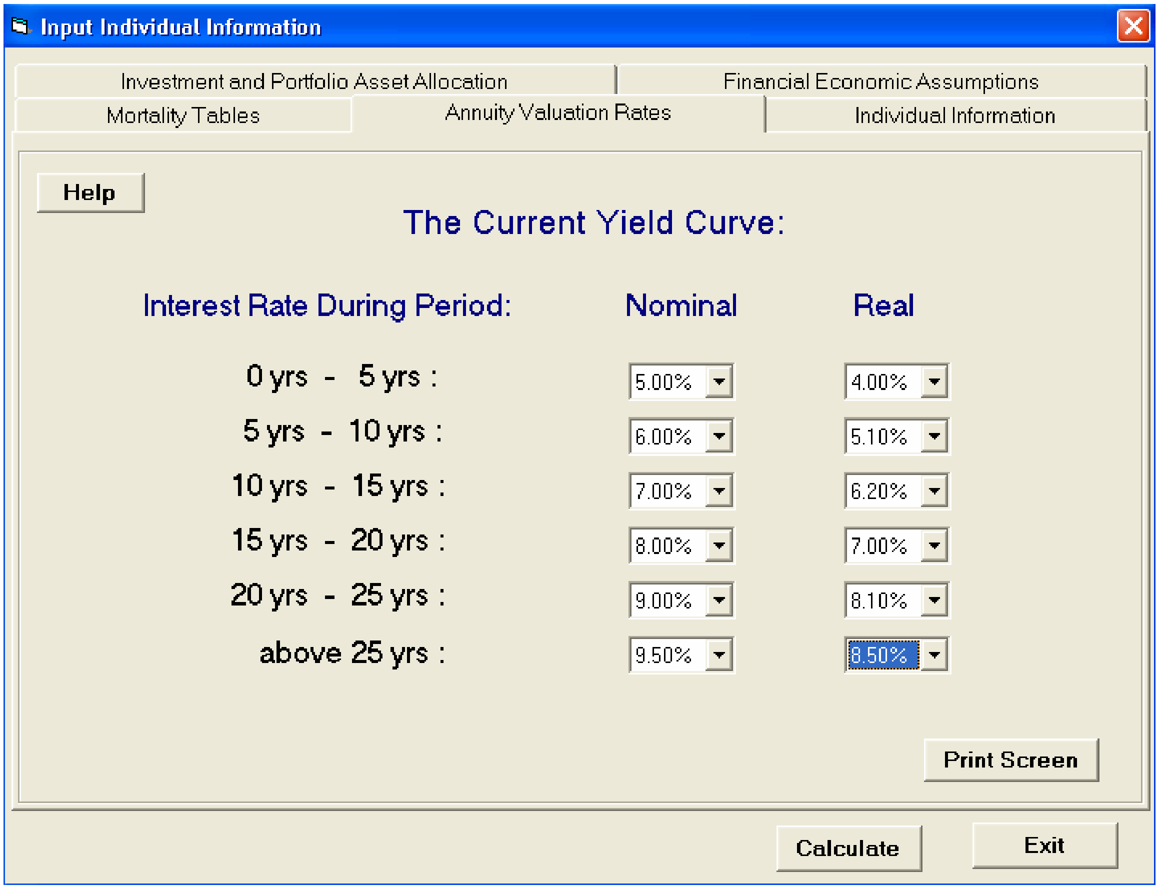
In this program, we allow the user to specify two different mortality tables in the *Mortality Tables* panel: one for annuity calculations and one for retirement ruin probability calculations. For more information on the table alternatives provided, please consult the SoA website (http://www.soa.org).

Both mortality tables may be projected forward to reflect mortality improvements. The methodology behind the table projections can be summarized as follows: a mortality table consists of a sequence of one-year mortality rates denoted by q_x and a projection (mortality improvement) scale denoted by g_x. The resulting projected conditional probability of survival is denoted by:

$$({}_n p_x) := \prod_{i=0}^{j} (1 - q_{x+i}(1 - g_{x+i})^i) \prod_{i=j+1}^{n-1} (1 - q_{x+i})$$

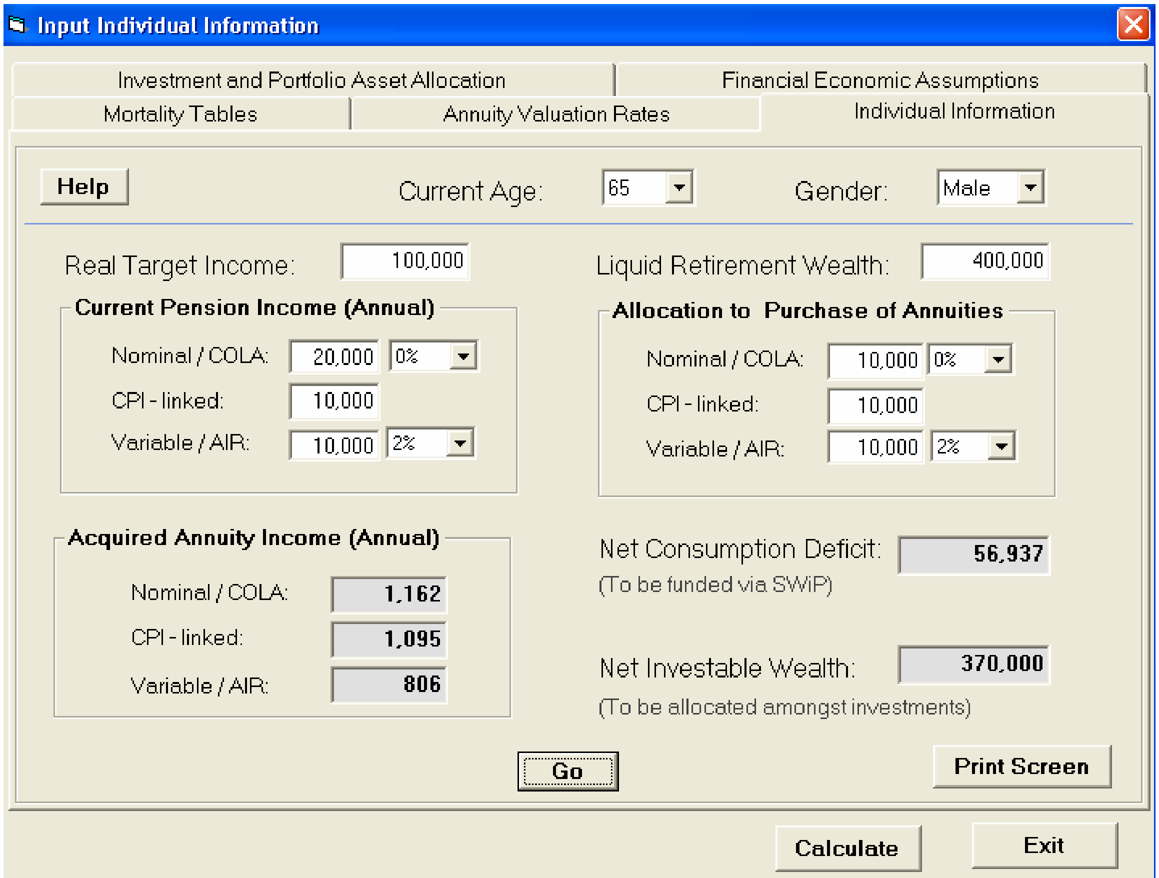
Where x is the current age, n is the number of survival years, and j is the number of projection years. For example, if j=n-1 (signifying a full projection - where the table is projected each year), $q_{60} = 0.001$, $q_{61} = 0.002$, $q_{62} = 0.003$, $g_{60} = 0.010$, $g_{61} = 0.015$, and $g_{62} = 0.020$, then the conditional probability of survival from age 60 to age 63 is:

$$(1 - 0.001(1 - 0.01)^0)(1 - 0.002(1 - 0.015)^1)(1 - 0.003(1 - 0.020)^2) = 0.99415$$



Annuity Valuation Rates

In the *Annuity Valuation Rates* panel, the program allows the user to specify a full term structure of interest rates (or a yield curve, which captures the relationship between interest rates of default-free bonds for different maturities) for the pricing of real (adjusted for inflation to be expressed in today's dollars) and nominal life annuities. It should be noted that the program "breaks" the yield curve into buckets of five years each, and discounts all payments during the given period at the specified rate. These numbers should be considered to be zero-coupon (not YTM) rates and are on an after expense and load basis. The program also assumes inflation by subtracting the real yield curve from the nominal yield curve, specified in this panel. Thus, since the two curves must be specified, inflation is not a stochastic variable in the program. The program does not account for unexpected inflation and thus, the inflation variable used in inflation scenario testing, for example, is limited to the value of the difference between the two yield curves.



Individual Information

The panel entitled *Individual Information* gathers the necessary information that will be used to assess the individual's financial future. The input of the following information is required:

- Current age – Ages 50 - 90 can be entered
- Gender
- Real target income - This is the desired level of real income, consisting of income from all sources, to be received throughout the retirement years.
- Liquid Retirement Wealth – This represents the amount of financial assets currently available for investment.

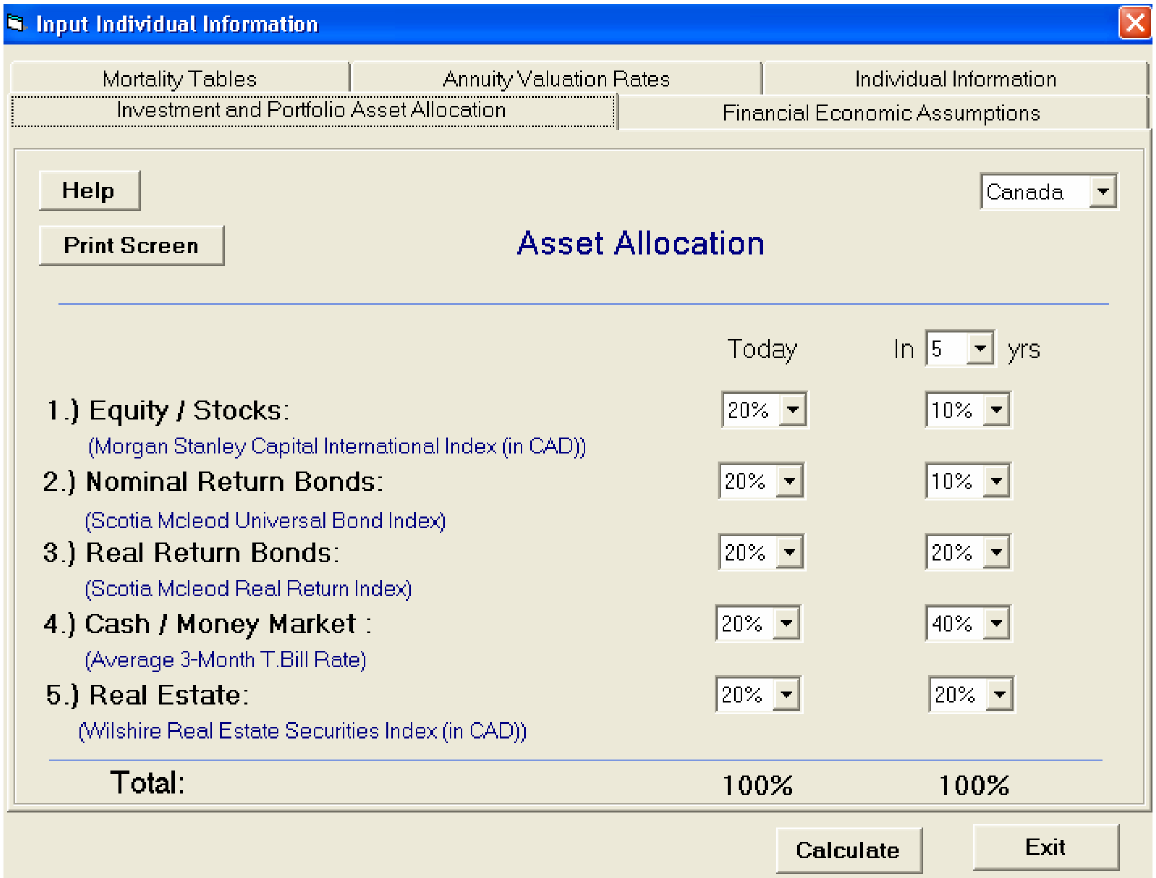
- Current Pension Income – All sources are to be entered. The program allows three types of entries of annual income: Nominal, CPI- Linked, and Variable. For Nominal annuities, the Cost of Living Adjustment ("COLA") rate is entered, and for Variable annuities, the Assumed Interest Rate ("AIR") is entered. As an example of current pension income, CPI – linked pension income may result from social security benefits and other public pension systems.
- Allocation to Purchase of Annuities – This section represents the amount of Liquid Retirement Wealth that has been allocated to purchase annuities. Similar to current pension income, this section allows three types of entries: Nominal, CPI-linked, and Variable. The total purchase price of each annuity type is entered, as well as the COLA Rate for Nominal Annuities and AIR Rate for Variable Annuities.

Once all of the above information has been entered, the user is ready to click the "Go" icon. Upon clicking this icon, the program calculates the remaining items in the panel. This feature can be used repeatedly on this panel to adjust the calculated values to the ultimate desired levels or for scenario testing. The following items are calculated:

- Acquired Annuity Income – This represents the annual income from each annuity type entered in the allocation to purchase of annuities section. The annuity income is determined based on the selected mortality table and term structure of interest rates, as well as the selected Assumed

Interest Rate (AIR) or the Cost of Living Adjustment rate (COLA) (depending on the type of annuity selected).

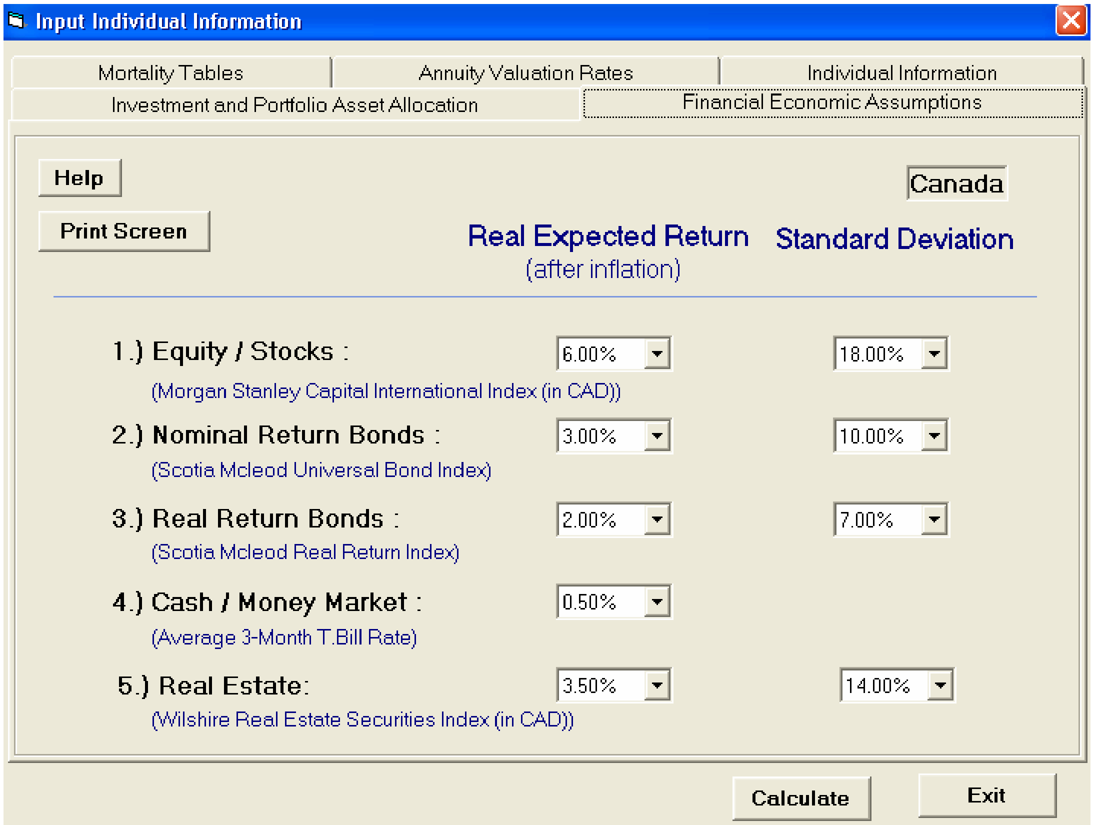
- Net Consumption Deficit - This represents the Real Target Income less the sum of Current Pension Income plus Acquired Annuity Income. It will be funded via a Systematic Withdrawal Plan (SWIP). The Systematic Withdrawal Plan will be financed using the Net Investable Wealth (NIW) or financial assets which have not been allocated to the purchase of payout annuities.
- Net Investable Wealth (NIW) – This represents the originally entered Liquid Retirement Wealth less the funds specified in the Allocation to Purchase of Annuities section.

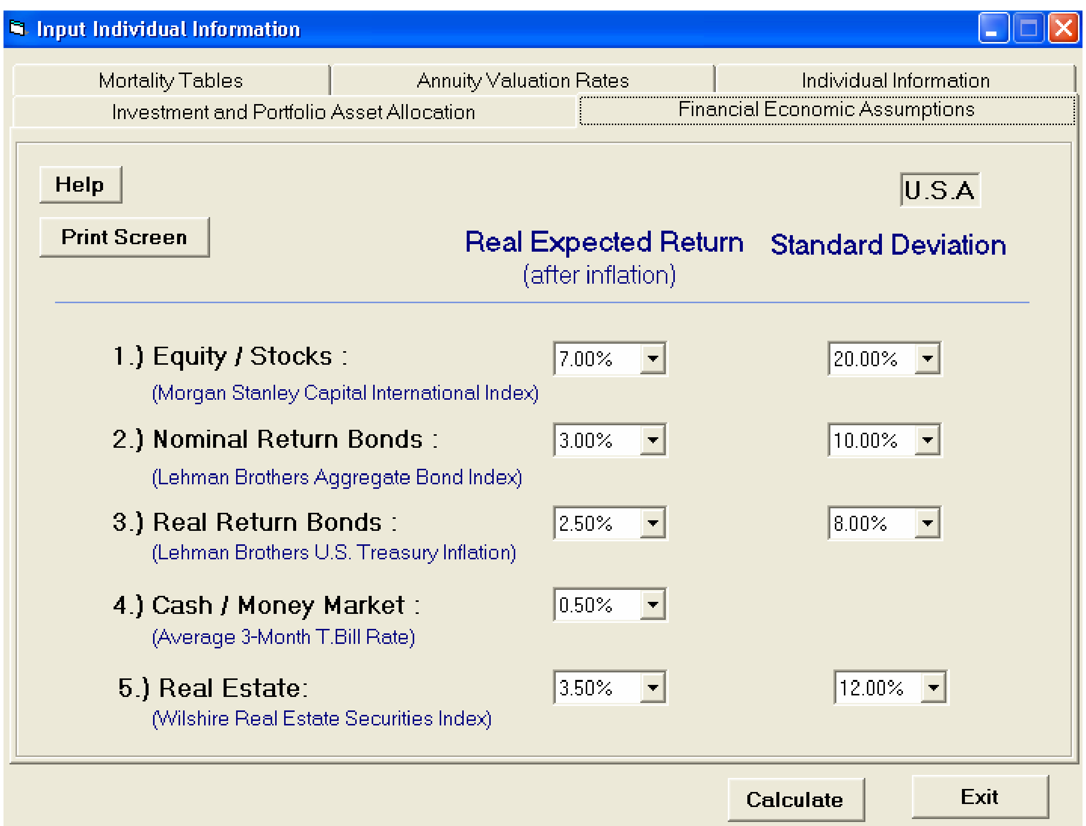


Investment and Portfolio Asset Allocation

In the *Asset Allocation* panel, the user must input the fraction of net investable wealth that will be invested in each asset class, which in total will form the diversified portfolio. Thus, the user should estimate the fraction of financial investments, including mutual funds and variable annuities that are equity-based, bond-based, etc. Please note that the sum of percentages of wealth invested in the asset classes must equal 100. Similarly, the program can account for the possible case in which the individual plans to change this allocation strategy several years into the future, in the adjacent column. The program treats this reallocation of assets, using a static reallocation method.

The asset classes that can be chosen for the purpose of investment have been represented here with the relevant indices which serve as benchmarks and reflect the current average behavior of the asset class. For example, the Morgan Stanley Capital International Index is a market capitalization weighted index tracking numerous equity markets across the world. Alternatively, the Lehman Brothers Aggregate Bond Index is generally representative of government bonds, good quality corporate debt, and mortgage or asset backed securities in the USA. Selecting Canada or the USA in this panel will display the relevant benchmarks for these countries.


Input Individual Information
Mortality Tables
Annuity Valuation Rates
Individual Information
Investment and Portfolio Asset Allocation
Financial Economic Assumptions
Help
Print Screen
Canada
Real Expected Return
(after inflation)
Standard Deviation
1.) Equity / Stocks :
(Morgan Stanley Capital International Index (in CAD))
6.00%
18.00%
2.) Nominal Return Bonds :
(Scotia Mcleod Universal Bond Index)
3.00%
10.00%
3.) Real Return Bonds :
(Scotia Mcleod Real Return Index)
2.00%
7.00%
4.) Cash / Money Market :
(Average 3-Month T.Bill Rate)
0.50%
5.) Real Estate:
(Wilshire Real Estate Securities Index (in CAD))
3.50%
14.00%
Calculate
Exit


Input Individual Information
Mortality Tables
Annuity Valuation Rates
Individual Information
Investment and Portfolio Asset Allocation
Financial Economic Assumptions
Help
Print Screen
U.S.A
Real Expected Return
(after inflation)
Standard Deviation
1.) Equity / Stocks :
(Morgan Stanley Capital International Index)
7.00%
20.00%
2.) Nominal Return Bonds :
(Lehman Brothers Aggregate Bond Index)
3.00%
10.00%
3.) Real Return Bonds :
(Lehman Brothers U.S. Treasury Inflation)
2.50%
8.00%
4.) Cash / Money Market :
(Average 3-Month T.Bill Rate)
0.50%
5.) Real Estate:
(Wilshire Real Estate Securities Index)
3.50%
12.00%
Calculate
Exit

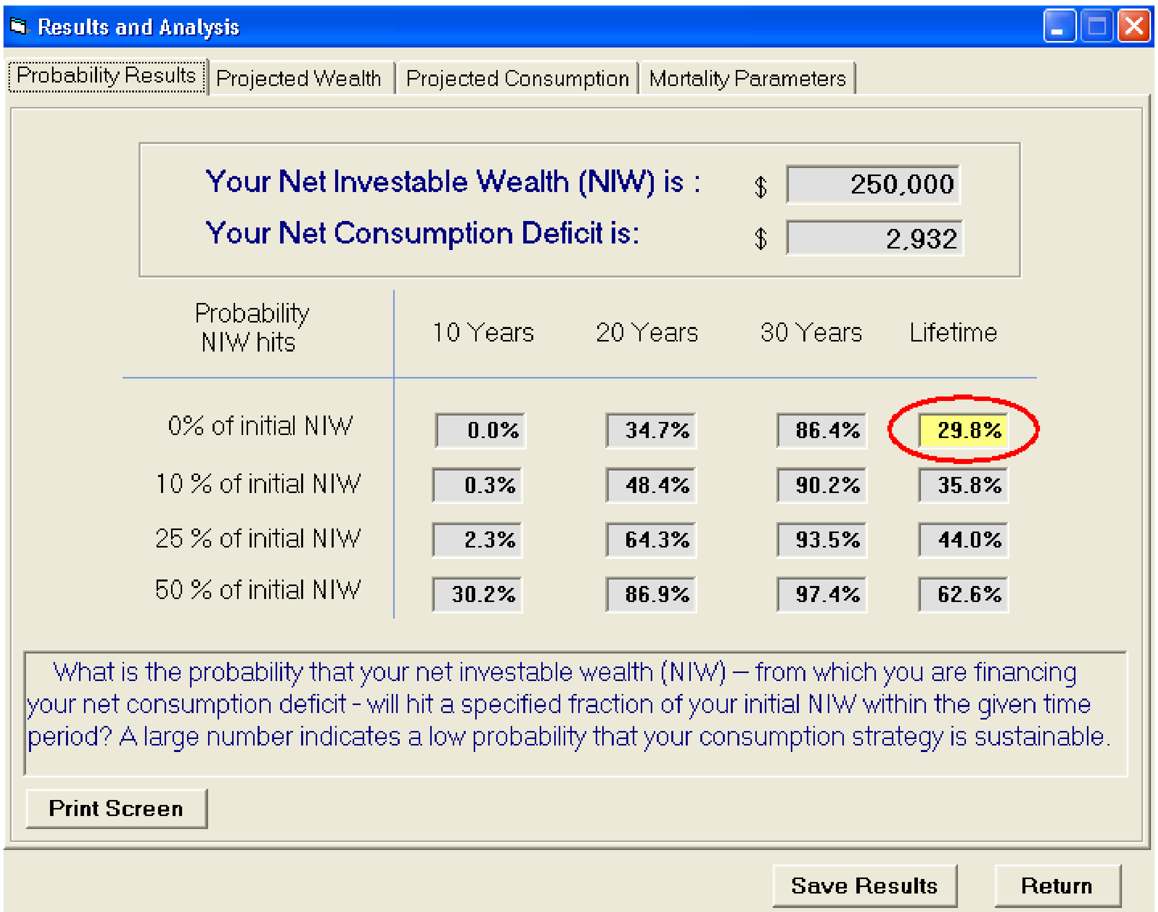
Financial Economic Assumptions

The purpose of the *Financial Economic Assumptions* panel is to gather information about the performance of each asset class in order to project the performance of the invested portfolio as a whole, which will serve as a crucial component in determining the retirement ruin probability. The real expected return of each index reflects the estimated return for each asset class, based on a probability distribution curve of the possible rates of return, adjusted for inflation. Next, the assumed standard deviation figures should be inputted such that they reflect the forward-looking volatility of the return (or the spread of returns around the average return) for the particular asset class. Currently, the default returns for these asset classes are long-term historical rates reported by Ibbotson Associates. Further, the correlation structure which determines the co-movement of the asset classes is hard-wired into the program. For an explanation of the impact of the correlation structure being hard-wired into the program on the results, please refer to Question 8 of the FAQ section for more details.

The *Financial Economic Assumptions* panels, illustrated immediately above, contain the default real expected returns and standard deviation parameters for the different asset classes for both Canada and U.S.A., and are provided as a reference to the user.

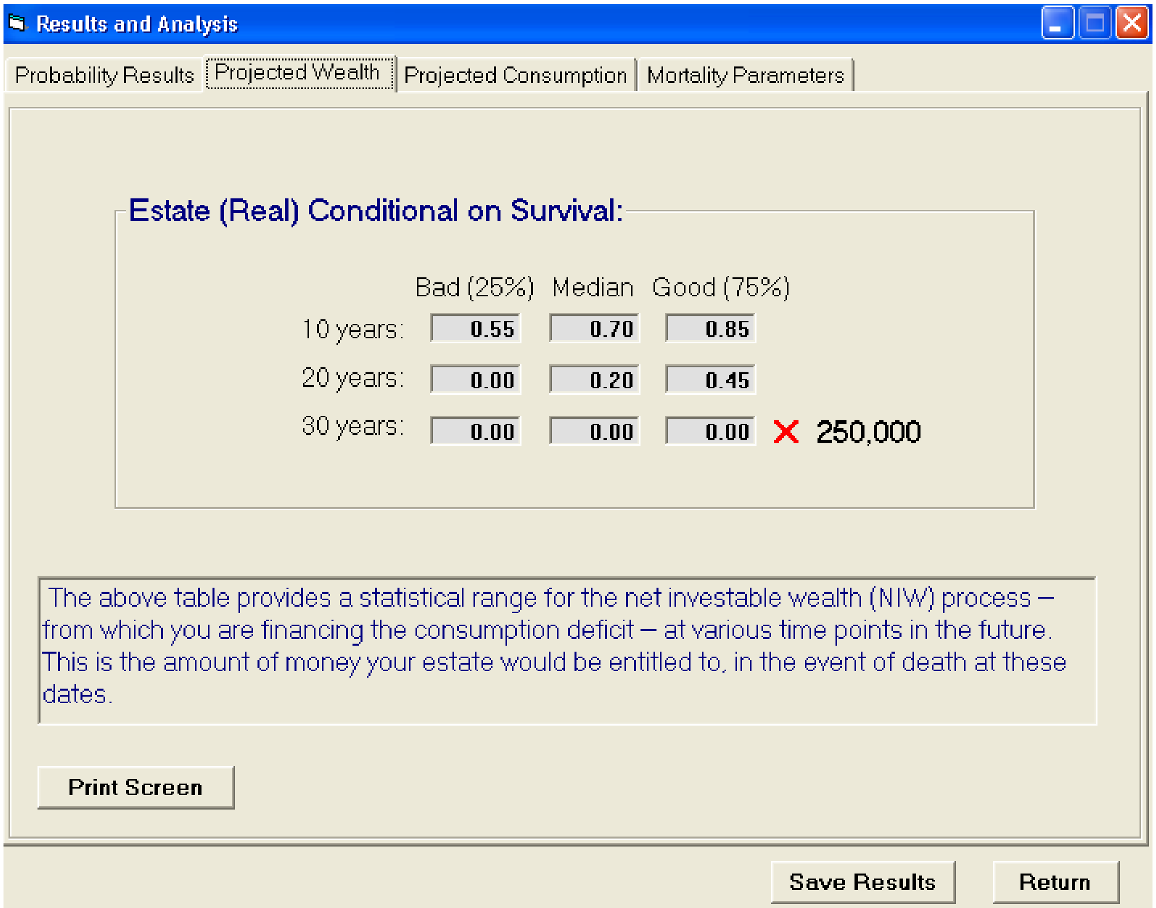
Part 2 – Results and Analysis

Once all the required variables are inputted, the program displays the *Results and Analysis* panels. Please note that in some cases, users may be required to wait between 30-90 seconds for the computations to be performed. The program allows the user to save the information from the Results and Analysis panels, in text format, by clicking on the "Save Results" icon on any one of the four Results and Analysis panels.



Probability Results

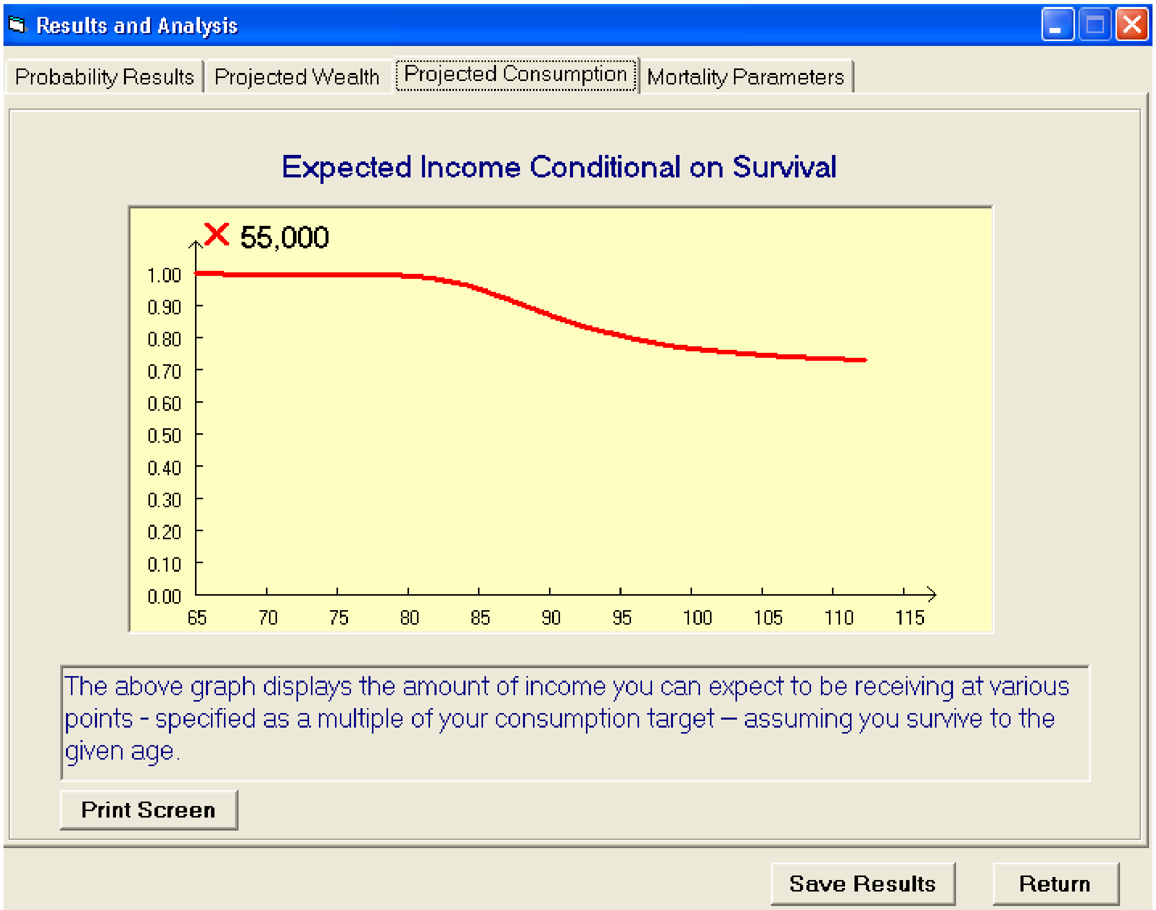
The output in the *Probability Results* section presents the end product of the program, or the consequences of the selected consumption and investment strategy. After taking into consideration the future life distribution of the individual, the likelihood of retirement ruin (that is, the probability of exhausting all available funds if no annuitization program is elected) within the lifetime is then calculated. A high probability, of course, signals a need for a strategy change and/or annuitization. To supplement this figure, the probabilities of depletion of the Net Investment Wealth to various levels (0%-50%) within 10, 20 and 30 years are also displayed. To review the underlying mathematical processes involved in attaining these results, please consult the accompanying technical appendix.



Projected Wealth

The *Projected Wealth* table presents the fraction of Net Investable Wealth that is projected several years into the future (10, 20 or 30 years). More specifically, this is the amount of money that the individual's estate would be entitled to, should death occur at these times (ignoring taxation). All values shown in the panel are on a real basis. The chart consists of three columns that represent bad, median and good case scenarios, where each is associated with a different probability (25%, 50%, and 75%) of the NIW, valuing up to a given fraction of the original NIW at the future times. These fractions are the output of the chart cells. Note that there is a 25% probability that the wealth will be less than "bad" and a 25% probability of the wealth being better than "good."

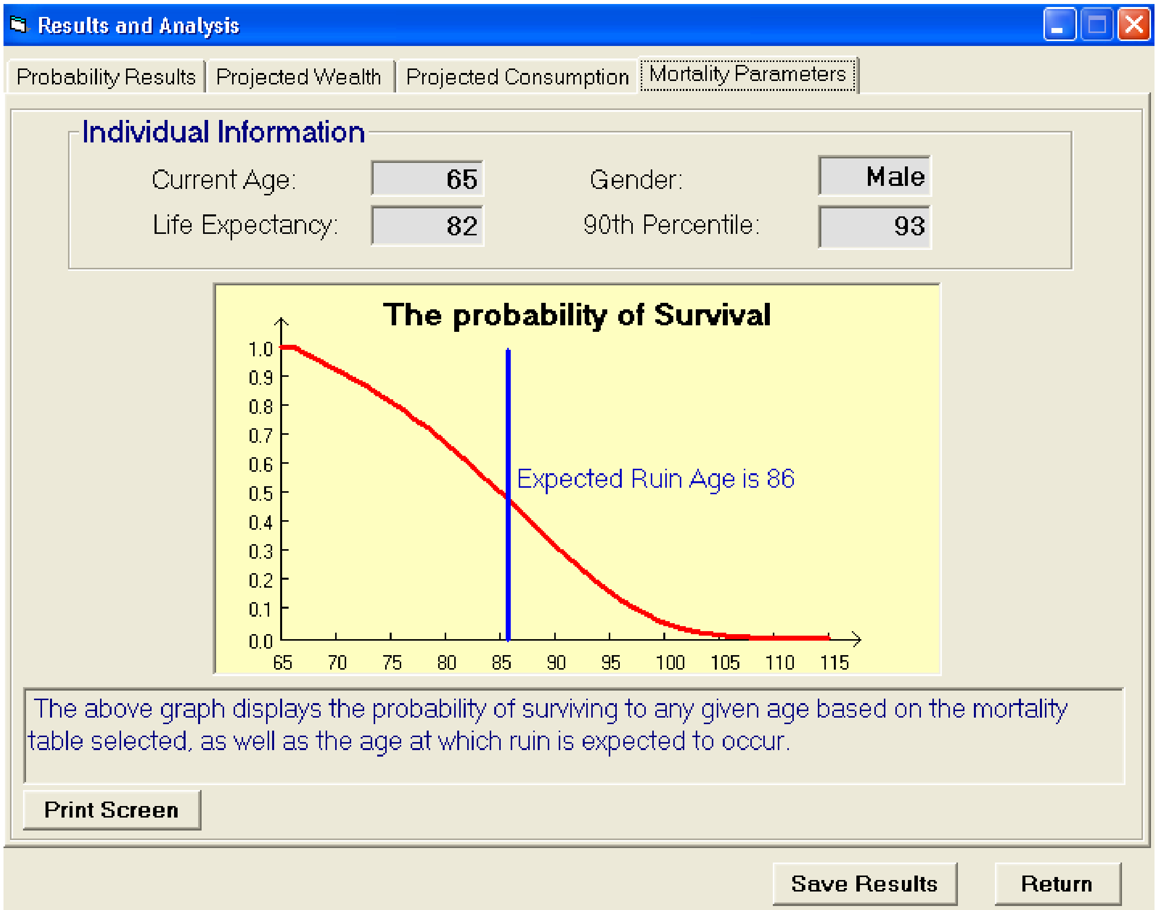
Here are some examples of how to read and interpret these numbers. In the above panel, 20 year results indicate that in 50% of future scenarios – i.e. with 0.50 probability -- NIW will be greater than 0.20 times $250,000 (=$50,000). This is the definition of median. 50% of the time NIW will be greater than $50,000 in twenty years and 50% of the time NIW will be less than $50,000. In 30 years, however, the median outcome is 0.0 times $250,000 which is obviously zero. Likewise, in 10 years time, there is a 25% chance that NIW will be less than 0.55 times $250,000 (=$137,500). This implies that there is a 75% chance that NIW will be greater than $137,500. On the flip side, after 10 years, there is a 25% chance that NIW will be greater than 0.85 times $250,000 (=$212,500). The same interpretation applies for 20 and 30 years. The 'three zeros' in year 30 should be interpreted to imply that over the many possible future scenarios for NIW, the large bulk of them lead to ruin (or zero value). More precisely, 75% of the time, NIW will be zero. However – and this is key – there is a less than 25% chance that wealth might be greater than zero.



Projected Consumption

The *Projected Consumption* panel graphically illustrates the mean outcome of income the retiree can expect to receive at various points in the future. All values in the graph are on a real basis. We stress again, that mean and median are different statistical quantities. The mean is an average that is computed by multiplying probabilities by outcomes while the median is the 50% mark at which half the outcomes are better, and half the outcomes are worse. *In our case, the median tends to be lower than the mean*. Thus, although the average or ‘mean’ consumption is displayed above, the median consumption level might be (much) lower at any given point in time. The mean might be distorted by a number of very large possible outcomes (i.e. in cases where the investor/retiree got lucky.) This is why caution is needed when examining the above picture and inferring from it the 50% mark at which money ‘runs out’.

The income level is expressed as a proportion of the Real Target Income level that was initially inputted, and survival is assumed for all ages on the horizontal axis. It should also be noted that this income pattern includes all sources of income, adjusted for inflation. This graph is especially useful in comparing investment and consumption plans with and without annuitization as it illustrates which investment strategy would allow the retiree to most closely achieve his/her Real Target Income goal.



Mortality Parameters

Finally, the *Mortality Parameters* panel summarizes statistics related to the individual's life distribution, and displays a probability function depicting the probability of survival to a given age. Superimposing the curve is a vertical line, which is the mean (*not* median or 50%) age at which ruin is expected to occur. In general, the panel graphically illustrates the risk associated with the current investment/consumption strategy when lacking an annuitization component. It is important to note, once again, that ruin does not imply zero income or starvation, rather it signifies that liquid wealth has been exhausted and all that remains is pension and annuity income, if any.

Part 3 - Case Scenarios

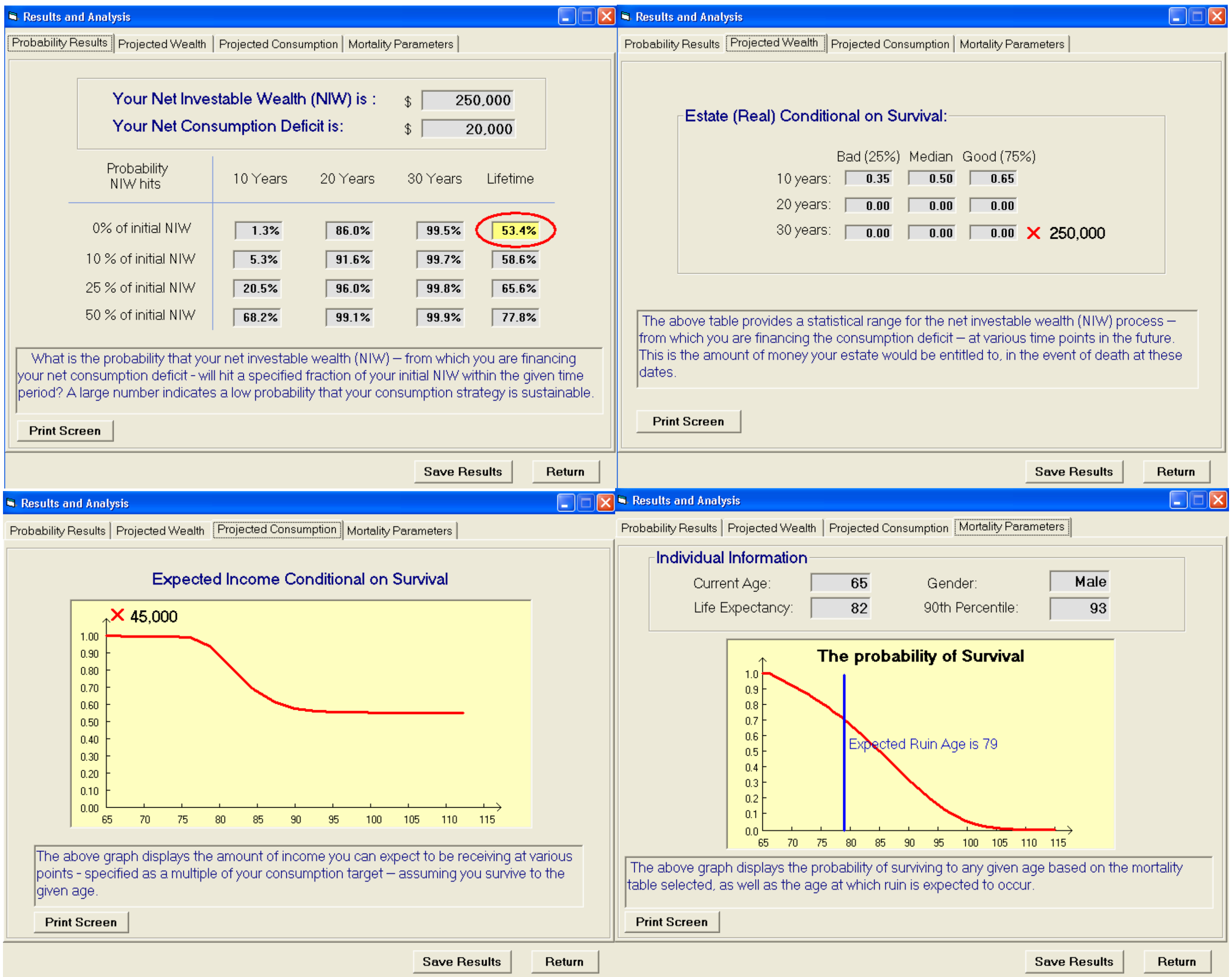
The purpose of the ensuing examples will be to contrast the outcome of an investment and consumption strategy that involves no annuitization with the outcomes of strategies involving varying allocations to immediate annuity purchases. Four examples will be developed, in which: 1) no payout annuities are purchased; 2) roughly half of liquid wealth is allocated to variable payout annuity purchase; 3) a large portion of liquid wealth is allocated among the three types of payout annuities; 4) the purchase of payout annuities is maximized. For all four examples, the following parameters are assumed: default mortality table selections, with no projections and the following real expected returns and standard deviation for the asset classes: Equity/Stocks: 7% return, 20% s.d.; Nominal return bonds: 3% return, 10% s.d.; Real return bonds: 2.50% return, 8% s.d.; Cash/money market: 0.50% return; Real estate: 3.50% return, 12% s.d. As well, a level term structure with a rate of 3% (real) and 5% (nominal); and an even distribution among the asset classes is assumed, with 20% in each category. The following inputted variables are also held constant throughout the examples:

Gender: Male
Age: 65
Liquid Retirement Wealth: $250,000
Real Target Income: $45,000
Pre-existing CPI linked annual pension income: $25,000

Case # 1: No purchase of payout annuities

- Only current source of income is the CPI linked annual pension income
- Resulting net consumption deficit = $20,000
- Net investable wealth of $250,000 is allocated among the 5 asset classes, and will finance the net consumption deficit through a systematic withdrawal plan (SWiP)

Results and Analysis: Case #1



Panel: *Probability Results*

- There is a 53.4% probability that the Net Investable Wealth will be entirely depleted within the individual's life time as a result of discretionary withdrawals of $20,000 and the performance of the diversified portfolio.
- Further into the future (10, 20, 30 years), the chances of the portfolio value depletion increase. For example, whereas within 10 years there is a 5.3% chance that the portfolio will be reduced to only 10% of the original net investment wealth, the chances drastically increase to 91.6% within 20 years. Assuming the individual survives 30 years (exceeding current life expectancy),the portfolio has a 99.7% probability of falling below 10% of the original NIW.

Panel: *Projected Wealth*

- Within 10 years there is a 75% chance that the portfolio value will fall to 65% (or below) of the NIW or $162,500. In the pessimistic scenario, there is a 25% probability that the value of the invested wealth will only measure up to $87,500.

Panel: *Projected Consumption*

- The curve demonstrates that the retiree's income target of $45,000 will only be met for a short period of time. Beyond this period, the negative slope of the line shows that the annual income will decrease as the portfolio value decreases, until the invested funds are exhausted. Then, only employment and government pension income of $25,000 will be received.

Panel: *Mortality Parameters*

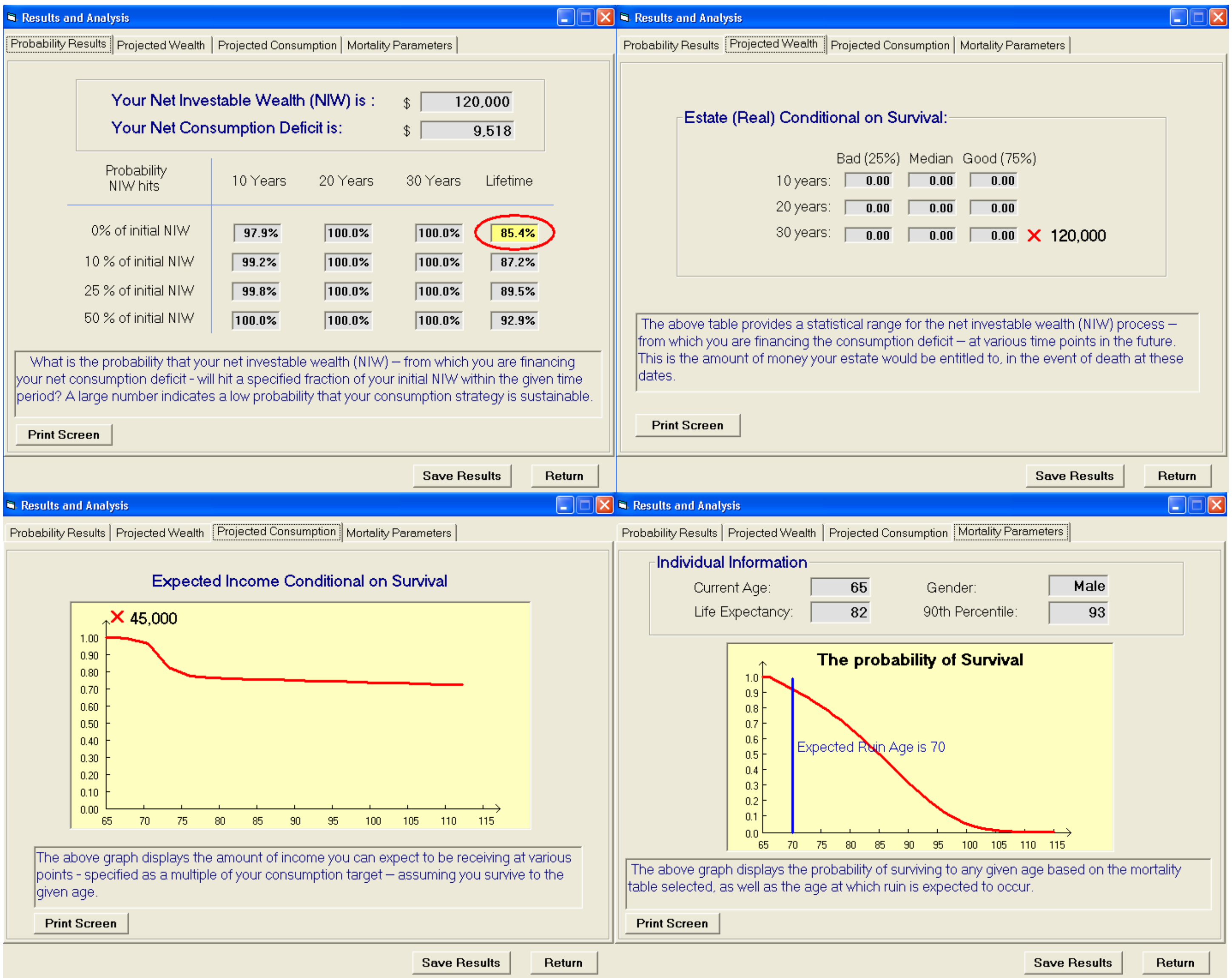
- The age by which the net investable wealth is expected to fall to zero, is 79.
- Based on the projection of life expectancy of the 65 year old male to be 82, and time of ruin to be the age of 79, there is a three year gap between the two events.
- After the time of ruin, the individual is expected to receive only pension income of $25,000 (adjusted for inflation).

Summary: The expectation of ruin in this scenario is quite high. If the retiree survives several years into the future, he will not be able to receive his annual real target income. Furthermore, should he pass away in ten years, there is a 75% chance that the wealth that his estate will be entitled to, will be reduced to less than 65% of the original funds. Should death occur in 20 years, there is a 75% probability his estate will not receive any fraction of his originally invested wealth.

Case # 2: Purchase of payout annuities

- Additional source of income: allocate a portion of liquid retirement wealth - $130,000 - to the purchase of variable payout annuities, selecting an assumed interest rate of 2%.
- Resulting net consumption deficit = $9,518
- Remaining net investable wealth = $120,000

Results and Analysis: Case # 2



Panels: *Probability Results and Projected Wealth*

- The original net investable wealth is expected to diminish to zero much sooner, since it is a much smaller initial sum than the NIW of Case 1, and will be reduced faster by discretionary withdrawals and portfolio losses.

Panel: *Projected Consumption*

- The retiree's $45,000 income target will only be met for a short period. Beyond this, annual income will decrease until it is comprised only of pension and annuity income.
- After the portfolio funds are exhausted, the combined pension and annuity income will allow the retiree to receive an annual sum approaching 75% of his target income.

Panel: *Mortality Parameters*

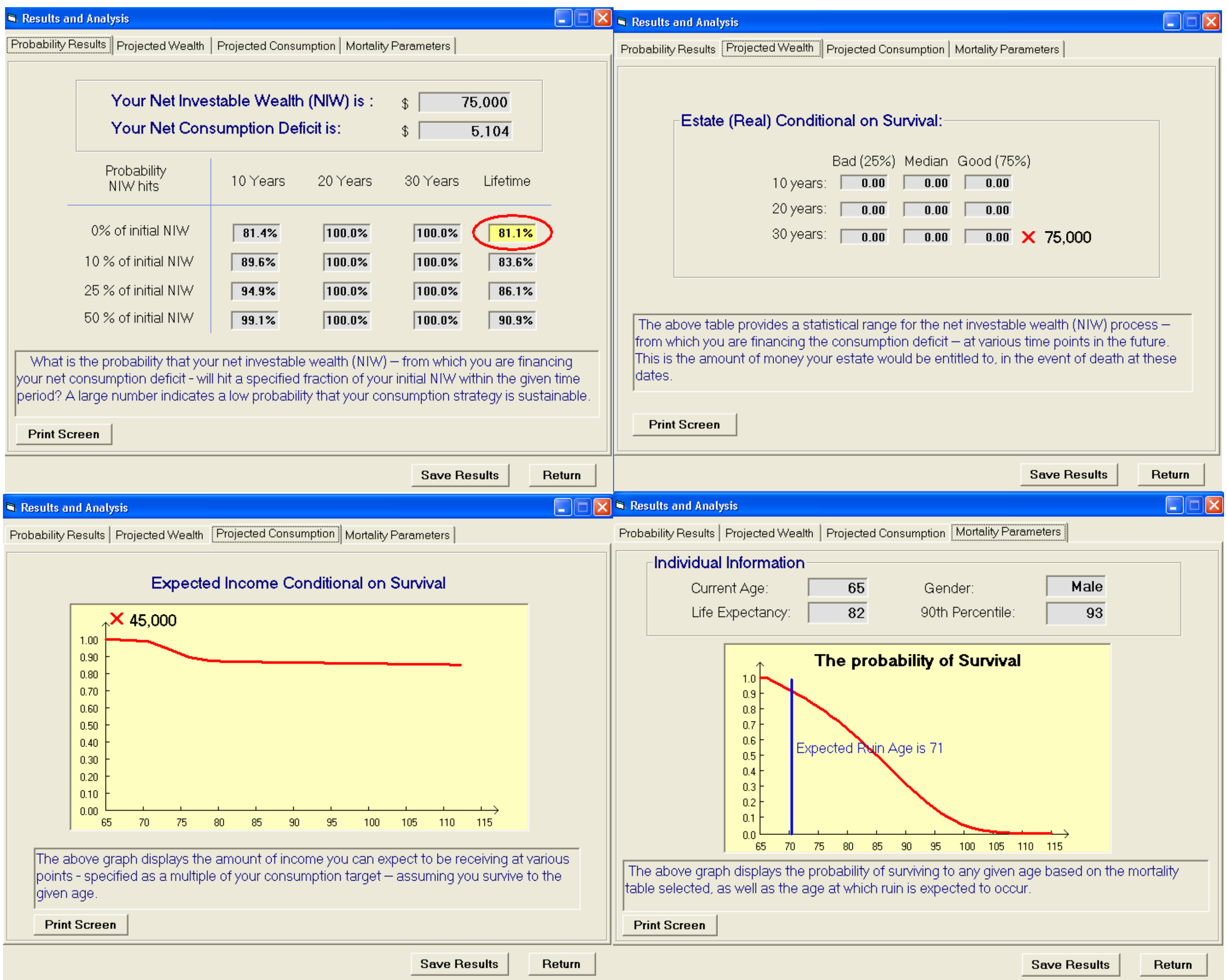
- The age by which the net investable wealth is expected to fall to zero, is 70.
- Between the life expectancy and projected point of ruin - which are 82 and 70 years of age, respectively - there is a 12 year gap.
- Following ruin, the individual is expected to receive pension and annuity income.

Summary: Wealth is expected to quickly decrease with age; however, income provided by the payout annuity finances a portion of the consumption deficit. Although the pattern of projected income also experiences a drop (as in Case 1), it remains closer to the target income.

Case # 3: Increased purchase of payout annuities

- Additional source of income: allocate an increased portion of liquid retirement wealth to the purchase of variable payout annuities:
  1) $50,000 in nominal payout annuities (adjusted 2% annually for cost of living).
  2) $60,000 in CPI-linked payout annuities.
  3) $65,000 in variable payout annuities (selecting an assumed interest rate of 2%).
- Resulting net consumption deficit = $5,104
- Remaining net investable wealth = $75,000

Results and Analysis: Case # 3



Panels: *Probability Results and Projected Wealth*

- The original net investable wealth is expected to quickly diminish to zero, with high probability, due to its small starting size.

Panel: *Projected Consumption*

- The retiree's $45,000 income target will only be met for a very brief period. Beyond this, portfolio funds will fall in value and annual income will be comprised only of pension and annuity income, which in sum result in an annual payment approaching 85% of the retiree's target income.

Panel: *Mortality Parameters*

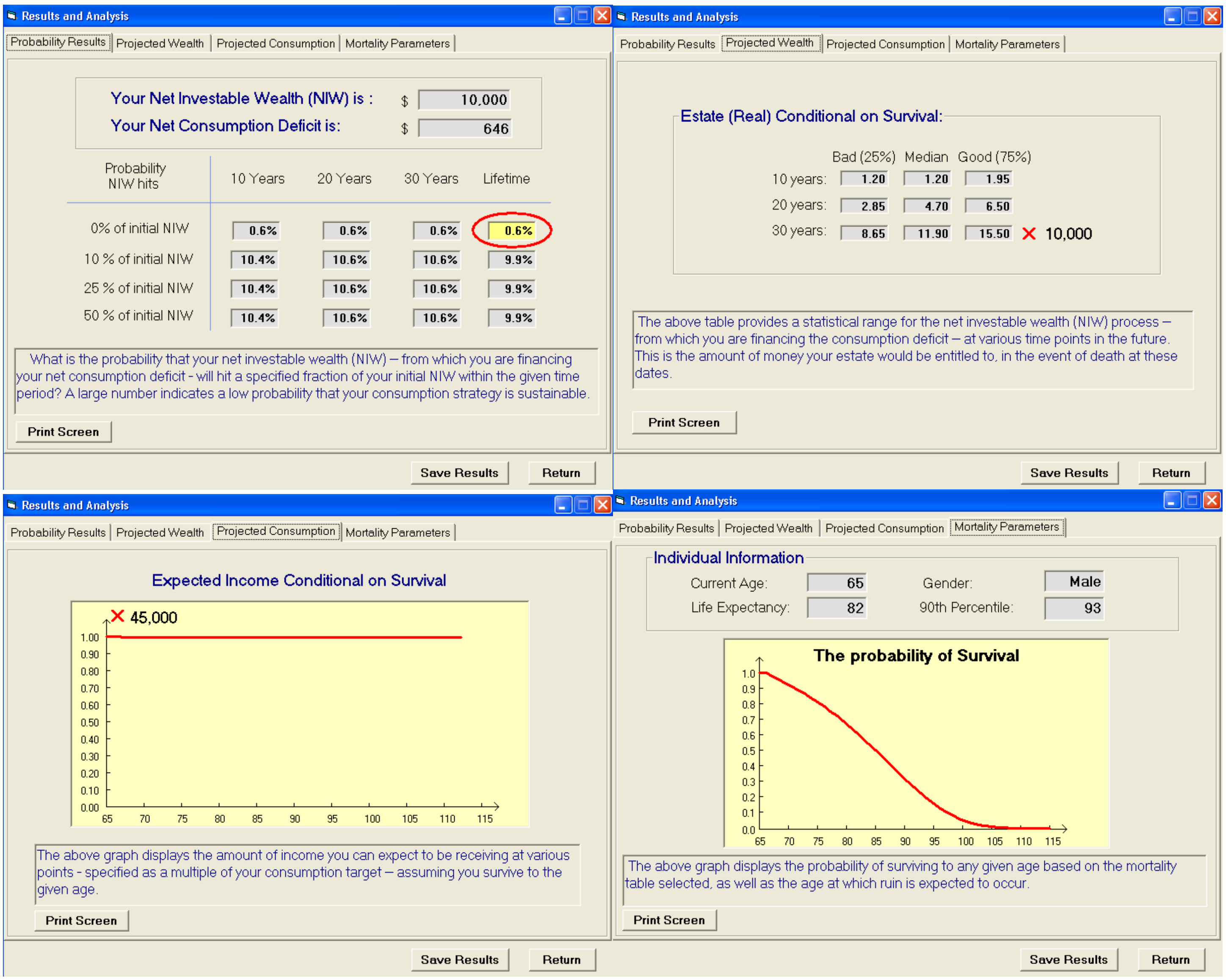
- The age by which the net investable wealth is expected to fall to zero, is 71.
- There is a gap between the life expectancy of the 65 year old male, which is 82 years and the projected point of ruin - age 71, of 11 years.
- The individual would be expected to receive only pension and annuity income after the point of ruin.

Summary: As in Case 2, wealth falls very quickly in value, leaving no funds to the estate in the event of death. However, Case 3 demonstrates a smoother expected income pattern, resulting from increased annuitization, which is closer to the target income.

Case # 4: Maximized purchase of payout annuities

- Additional source of income: allocate nearly all liquid retirement wealth to the purchase of nominal payout annuities:
  -$240,000 in nominal payout annuities (adjusted 3% annually for cost of living).
- Resulting net consumption deficit = $646
- Remaining net investable wealth = $10,000

Results and Analysis: Case # 4



Panels: *Probability Results and Projected Wealth*

- There is little probability that the original net investable wealth will decrease to the level of zero, since there will be little reliance on the systematic withdrawal plan for the purpose of net consumption deficit financing. Thus, the initial sum will largely remain intact, accumulating returns. For example, in 20 years, there is a 75% probability that the portfolio value will reach as high as 650% of the original net investable wealth.

Panel: *Projected Consumption*

- The graph illustrates that the investment strategy will allow the retiree to closely meet his $45,000 income target throughout the lifetime, regardless of the actual lifespan.

Panel: *Mortality Parameters*

- The panel reiterates the fact that the net investable wealth is not expected to fall to zero, throughout the lifetime.

Summary: In the case scenarios developed above, maximizing the amount of the net investable wealth allocated to the purchase of payout annuities allows the annuitant to better ensure that his real target income goal is met throughout retirement years. The strategy does however constrain the retiree's accessibility to his funds, should he require a larger than usual income at some point during his retirement.

As a result of the selected investment strategy, the wealth that the estate would be entitled to in the event of death would be larger than that of cases 1-3, in most scenarios.

It is important to recognize that the maximized annuitization case allowed the retiree to best reach his real target income and resulted in a higher level of wealth (in most scenarios) because the target income figure selected was relatively small in proportion to the initial liquid retirement wealth.

Frequently Asked Questions

1. *There has been criticism recently about using Monte Carlo Simulations (MCS) for retirement planning and estimating long-range probabilities. Does this Retirement Ruin (ReRu) algorithm use MCS?*

No. This program is based on analytic approximations and numerical solution techniques for Partial Differential Equations (PDE) and therefore does not require Monte Carlo Simulations of any kind.

2. *Does the algorithm use the actual mortality tables selected in the input panels or does it use an approximation to these tables?*

The algorithm requires a continuous law of mortality as input since death and/or ruin can occur at anytime during the year. Yet, most mortality tables are annualized. Therefore, once the user selects the relevant mortality tables and projection factors, the algorithm locates the best fitting Gompertz-Makeham curve to the dynamically projected table and then uses the implied parameters in all calculations.

3. *Does this mean that if annuity factors are computed using the discrete version of the actual tables, they will differ from the ones computed in the program?*

Yes, but only slightly. At worst there will be a 2% to 4% difference between our annuity factors and the ones you might compute using traditional (annual) payments. The justification for this assumption is that our ReRu algorithm is much more sensitive to changes in capital market parameters, and using actual dollar-for-dollar payments might give a false sense of precision.

4. *If the investor is more healthy or less healthy than the average retiree, can the program account for this?*

No. Not exactly. But there is a ‘trick’ that can easily be used to force the program to consider the investor to be healthier or less-healthy by modifying the projection scale on the mortality tables used for either annuity or ruin probability calculations. For example if he or she believes they can qualify for an impaired annuity – which would provide greater income during retirement – no projections would be selected. Vice versa, if the individual thought they were healthier than average and hence had a longer life expectancy, a longer (greater) projection period could be selected for the ruin calculations.

5. *The retiree has a surviving spousal benefit of 60% and the payout annuity she purchased has a 10-year payment certain. How can these details be inputted into the program?*

The short answer is that they cannot be. All of our annuities are structured as *life only* with zero term-certain and no survivor benefits. Clearly this is unrealistic, but is deliberately so. Our intention with the ReRu algorithm is to provide intuitive guidance on the impact of various asset allocation and annuity purchase decisions on the probability of retirement ruin. We hope the program will help the user develop a pedagogical intuition for the impact of various choices. This program is not intended as a comprehensive retirement planning tool. That said, even if we did allow these additional choices as inputs, the extra 'moving parts' in the program would cloud the insights that can be gleaned from the output probabilities and consumption paths.

6. *The program only allows five different investment choices for the net investable wealth (NIW) and variable payout annuity (VPA) category, yet a particular 401(k) and Variable Annuity allows the investor to choose between many more sub-accounts. Why is the program so restrictive?*

Although the range of investment choices in the program is admittedly narrow, the sub-accounts that comprise the choices for variable annuity investments can typically be classified under one of RetPro's asset classes because of their comparable characteristics.

7. *The program and manual often uses the term ruin, but in fact, even if the algorithm displays a large ruin probability, this is not necessarily bad news, is it?*

Correct. Ruin, in our model, is just net investable wealth hitting zero. But if there is sufficient annuity income, the retiree may still enjoy a long and prosperous retirement.

8. *What is the source of the default capital market parameters and the built-in covariance matrix that is hard-wired in the program?*

As a starting point we used the Ibbotson Associates yearbook (2003) parameters for the various asset classes. We then used a variety of smoothing procedures and decided to pick, what we consider to be, reasonable default parameters and a zero correlation matrix, except for a 50% correlation between real and nominal bonds.

The correlation matrix is hard-wired into the program, to ensure that the probability of financial ruin is not impacted by unreasonable assumptions the user may make about the correlation among the five asset classes (note, the matrix must be invertable). The program, however, leaves the remaining variables - expected return and standard deviation - as free parameters that can be manipulated. Therefore, the quality of the program's outputted results will depend on the soundness of the user's assumption about these two variables. The impact of changing these parameters can be seen via experimentation. But as one would expect, more 'bullish' assumptions will result in lower ruin probabilities and vice versa.

9. *Some earlier work provided guidance on the optimal age at which to annuitize. Is this information still available in the program?*

An earlier (beta) version of the program had a panel that computed the value of the option to delay annuitization – based on the recent work by Milevsky and Young (2004). However, the current and final version does not include this material. It was decided that this information was not within the scope of the intended use of the program.

10. *In some cases, the expected ruin age is quite far in the future, even though there is a very high probability NIW will hit zero within 30 years. How can these seemingly conflicting results be reconciled?*

By definition, the expected value of any random variable – such as the age at which NIW will hit zero – tends to be influenced by extreme events with small probabilities. The median, in contrast, is not skewed by extreme values, which is why the mean (expected) age of ruin will always be greater than the median age of ruin. This is the case for all positive numbers. Therefore, it is quite feasible that the probability of the NIW hitting zero is quite high at any given horizon, yet the expected ruin time is far in the future.

11. *Why does the graph in the* Projected Consumption *panel not show a sharp drop at the same age that ruin is expected to occur, which is shown in the* Mortality Parameters *panel?*

Remember that mathematical expectations – weighting outcomes by their probabilities – are tricky quantities. Note that consumption and income levels can vary widely over time and the projected consumption is only an expectation. There is a chance that consumption will be (much) higher, and also a chance that it will be (much) lower. The actual spread of consumption will have an important impact on the ruin time. In addition, there is a non-linear relationship between consumption and wealth, and although zero wealth implies zero consumption and vice versa, the distribution of these quantities are quite different.

12. *Is there additional information about the underlying algorithm and similar research papers?*

References:

The following research papers and manuscripts explain some of the theoretical ideas in greater detail.

G. Kingston, S. Thorpe. "Annuitization and Asset Allocation with HARA Utility," April, 2004, working paper.

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M. A. Milevsky, "Optimal Asset Allocation Towards the End of the Life Cycle: To Annuitize or Not to Annuitize?", Journal of Risk and Insurance, Vol. 65(3), September 1998, pg. 401-426.

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M. A. Milevsky "Asset Allocation and Annuitization", (with V. R. Young), January 2004, working paper available at www.ifid.ca

M.A. Milevsky, "Ruined Moments in Your Life: How Good Are the Approximations? (with H. Huang and J. Wang), Insurance: Mathematics and Economics, to appear in 2004.

M.A. Milevsky, "Ruin Minimizing Annuitization Strategies", (with K. Moore and V. Young), working paper available at www.ifid.ca

M. A. Milevsky, "The Real Option to Delay Annuitization: It is not Now-or-Never", (with V. R. Young), 2003 under advanced review Management Science.

R. Maurer, "Betting on Death and Capital Markets in Retirement: A Shortfall Risk Analysis of Life Annuities verses Phased Withdrawal Plans (with I. Dus and O. Mitchell)," April 2004, working paper.

Technical Appendix

Our financial market is modeled as a geometric Brownian motion (GBM) with drift and diffusion parameters denoted by $\{\mu, \sigma\}$ which, in turn, are the properly weighted averages of the individual asset's expected return and volatility (a.k.a. standard deviation) parameters. These parameters can be time dependent, and the program allows for one change after a pre-specified number of years. The process followed by one unit of a variable account is (where X is the Random Value of a unit and B is Brownian motion):

$$dX_t = \mu X_t dt + \sigma X_t dB_t, \;\; X_0 = 1. \qquad \text{(eq.1)}$$

We are currently at time $t = 0$, with an individual aged x, who has liquid and investable wealth denoted by the symbol W. Our retiree receives three types of (pre-existing) pension annuity income: a variable life annuity, a real (inflation adjusted) life annuity and a nominal life annuity. They are denoted by c^v, c^r, c^n respectively. These payout (a.k.a. immediate) life annuities have additional parameters governing their dynamic behavior. The variable annuity requires the following as input: an assumed investment return (AIR) denoted by the letter h, the nominal annuity requires a cost-of-living adjustment parameter denoted by the letter l, which can also be treated as an escalating annuity at this rate per annum. The retiree then decides how much additional wealth to dedicate to annuity purchases. Once that decision is made, the initial value of net investable wealth (NIW) is defined according to:

$$W_0 = W - c_0{}^n \int_0^\infty ({}_t p_x) e^{(l - R_t{}^n)t} dt - c_0{}^r \int_0^\infty ({}_t p_x) e^{(-R_t{}^r)t} dt - c_0{}^v \int_0^\infty ({}_t p_x) e^{(-h)t} dt \qquad \text{(eq.2)}$$

where the symbol ($R^r{}_t$) denotes the continuously compounded real yield curve and ($R^n{}_t$) denotes the nominal yield curve. The intuition is as follows: the first integral in (eq.2) captures the per-dollar-factor for the nominal annuity that increases at the COLA rate. This pricing scheme discounts cash-flows based on the nominal curve ($R^n{}_t$). The second integral covers the real (inflation-linked) annuity and therefore discounts based on the real curve ($R^r{}_t$), while the third integral discounts based on the assumed interest rate (AIR) denoted by (h).

As time evolves, the various annuity income flows – in real terms -- according to the following dynamics:

$$c_t{}^n = (c_0{}^n + c^n)e^{lt-\pi_t}, \quad c_t{}^r = (c_0{}^r + c^r), \quad c_t{}^v = (c_0{}^v + c^v)X_t e^{-ht}e^{-\pi_t}. \quad \text{(eq.3)}$$

The intuition is as follows: the nominal annuity will grow at the COLA rate but must then be deflated by the cumulative inflation factor, defined by (π_t). The real annuity will, by definition, maintain real value over time. The variable annuity will fluctuate based on the performance of the market account (X_t), but is then deflated by the AIR value (h) and finally deflated by the cumulative inflation factor (π_t). In our current model, the inflation factor is defined by the integral of the difference between the nominal and real interest rate curves in the following manner:

$$\pi_t = \int_0^t (R_s{}^n - R_s{}^r)ds. \quad \text{(eq.4)}$$

If the difference between the two curves is a constant, so that $i = R_s{}^n - R_s{}^r$, then the inflation deflator will be a simple $\pi_t = it$. Note that our ability to combine similar cash-flows in (eq.3) is the main reason we restricted our model to AIR and COLA values that are the same for acquired and pre-existing annuity income. Of course, any of the six values $\{c_0{}^n, c^n, c_0{}^r, c^r, c_0{}^v, c^v\}$ can be set to zero without any loss of generality. We now define a new variable:

$$c_t{}^a = c_t{}^n + c_t{}^r + c_t{}^v, \quad \text{(eq.5)}$$

which is the sum of all the annuity income flow that is received at any point in time, in real after-inflation terms. The difference between the annuity income flow $c_t{}^a$ and the target consumption $\hat{c}$, is defined by $c_t = \hat{c} - c_t{}^a$ and withdrawn from (or possibly added to) the discretionary wealth account. Thus, the dynamics of W_t will obey:

$$dW_t = dX_t - dc_t, \quad \text{(eq.6)}$$

The total amount consumed will be the target $\hat{c}$, if the wealth process $W_t > 0$ and $c_t^{\,a}$ if the wealth process $W_t = 0$. Our algorithm computes a number of relevant summary statistics; the first is the hitting probability, which is defined equal to:

$$H_2(y) := \Pr\left[\inf_{0\le t\le T} W_t = y \middle| W_0 = w\right], \qquad \text{(eq.7)}$$

where (T) is either a random time of death, or a fixed horizon. For example, the quantity $H_2(0)$ would capture the probability of ruin, which is hitting zero. We also define a related quantity:

$$H_1(y) := \Pr\left[W_T \le y \middle| W_0 = w\right], \qquad \text{(eq.8)}$$

which is the probability of the process being less than or equal to a level (y) at some point (T). Thus, if we are interested in the median value of net investable wealth in (T) years, then we compute $H_1^{-1}(0.5)$, which is the inverse of the probability evaluated at the value of 50%. In the same manner, $H_1^{-1}(0.25)$ gives us the 25th percentile.

Finally, the expected consumption at any future time (or age) will be computed according to the formula defined by:

$$H_2(0)E[c_T^{\,a}] + (1 - H_2(0))\hat{c}\,.$$

The formula represents the probability of ruin, multiplied by the expected annuity income flow, plus the probability of sustainability, multiplied by the target income level. In theory, one can also compute a standard deviation for consumption at any given age using the same idea.